

Scheduling of Next Generation Timetable Systems

Problem presented by

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Airbus

Executive Summary

The scheduling of future timetables is an important driver for aircraft sales and design considerations in future aircraft. Airline companies seek to maximise their profits through capturing passenger demand through the quality of service they offer, within which timetabling plays an important role.

In this study a hub and spoke system for medium haul travel is analysed, with particular reference to the time at which departing waves are set from the hub airport. Initially, an optimal wave time based only on the geography of the hub is considered. Subsequently, a model is developed which includes the constraints of market share and limited fleet size, and an example timetable produced. A final note is made about game theoretical aspects that might be considered in moving the work forward.

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Contents

1	Introduction	1
1.1	Background and scope	1
1.2	Overview of the methodology	3
2	General approach taken	3
2.1	Available data	3
2.2	Optimizing bank time in the P2P model	4
2.3	Connecting flights	7
2.4	Improved model	7
2.5	Competition and game theory	10
3	Conclusions	10
3.1	Remarks	10
3.2	Suggested further research	11
A	Appendix	12
A.1	Calculation of the Quality of Service Index	12
A.2	Timetable obtained from improved model for LHR	12
A.3	Nash equilibrium	12
	Bibliography	14

1 Introduction

1.1 Background and scope

- (1.1.1) Airbus provides consulting services to airlines seeking to grow and/or renew their profitability and therefore their need to purchase aircraft. Hence, the ability to provide airlines with analysis of optimum aircraft scheduling will increase the potential of Airbus to generate profit.
- (1.1.2) In the long term, Airbus is working on future aircraft designs to help mitigate the growing constraints to achieve sustainable development of the aviation industry. Understanding future aircraft schedule frameworks will enable Airbus to better understand the likely requirements of new aircraft.
- (1.1.3) The general problem can be stated as ‘compute a future network timetable and associated fleet plan that maximizes the airline’s profit under external constraints’. The problem contains a large number of parameters and hence some simplifications are proposed to make the problem manageable for the study group.
- (1.1.4) Initially the hub and spoke system will be considered for medium haul aircraft. The hub and spoke system regards a single airport as the hub for an airline’s operations with point-to-point (P2P) flights from the hub to local airports. In the hub and spoke system aircraft arrivals and departures occur in waves. A typical example is shown in Figure 1. In this system an aircraft will depart from the hub airport on one wave (or ‘bank’), and return 2,3,... waves later to the same airport. The time in between departure and arrival (labeled total turnaround time, T_{OD}) is required for flying to the destination, turnaround time at the destination (≈ 30 mins), flight back to the hub and finally reconfiguring the aircraft such that it is ready for departure from the hub again (≈ 45 mins). Ideally, one should consider both long haul and medium haul flights in a hub configuration. However, because there are relatively few long haul flights and carriers have a greater flexibility over the take-off/landing times of the long haul flights the two problems can be partitioned in the following way. The carrier can define the medium haul time table, and the long haul flights simply dovetail into this pattern by shifting their take-off times appropriately.
- (1.1.5) A number of industry parameters are used to evaluate the performance of an airline based on empirical analysis of past data. The market share M (proportion of passengers on a route that the airline will attract) is given by

$$M = \frac{Q_0}{Q_0 + \sum_{i=1}^N Q_i}, \quad (1)$$

where Q_0 is the Quality of Service Index (QSI) for the airline and Q_i

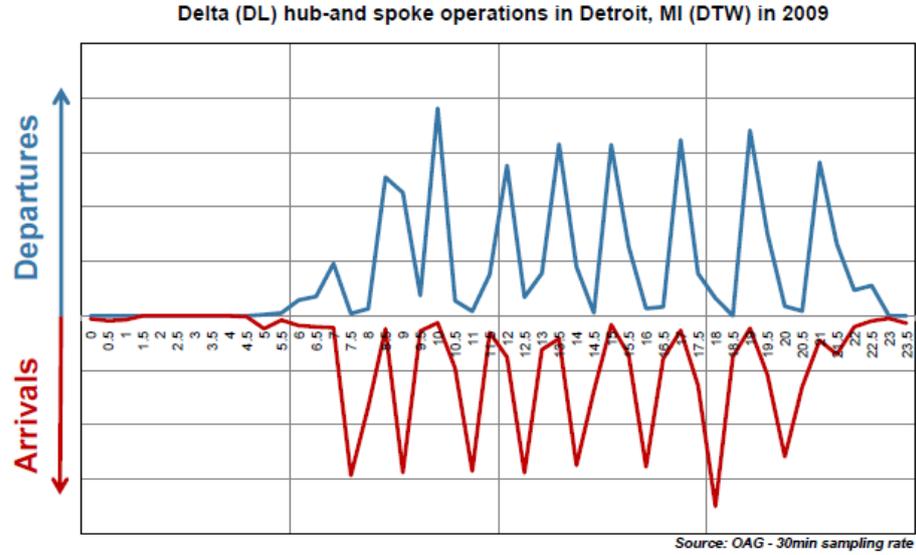


Figure 1: Arrivals and departures in a typical hub and spoke system.

($i = 1, \dots, N$) is the sum of the competitors' QSI (airlines offering the same route). The QSI reflects a passenger preference for aircraft type, number of stops, flight frequency, travel elapsed time, fare, time of day and day of week. The details of the coefficients to calculate QSI can be found in Appendix A.1. Interestingly, fare price is not included in the QSI model due to the lack of reliable data, however the price will clearly have an effect on whether a passenger will use a competitor's service or not.

(1.1.6) The overall objective of any schedule is to maximise profit

$$s = \sum_{OD} (P_{OD} \times f_{OD} - C_{OD}^{tot}), \quad (2)$$

where s is the profit, P_{OD} is the number of passengers on a particular route (origin to destination), f_{OD} is the average fare for the corresponding route, C_{OD}^{tot} is the total cost associated with that route. The true situation is often more complicated since national carriers can have political constraints requiring them to serve non-profitable routes but these are ignored for the purposes of this analysis.

(1.1.7) The relationship between passenger number and overall demand for a route is given by

$$P_{OD} = M_{OD} \times D_{OD} \quad (3)$$

where D_{OD} and M_{OD} are the absolute (passenger) demand and market share for a route respectively.

- (1.1.8) The cost of operation for a single aircraft, C_{OD} , is aircraft dependent and is a linear function of the distance for a particular route. It is comprised of the take-off, landing and navigation costs, fuel burnt during the flight, maintenance costs and crew costs. A fixed daily ownership cost for each aircraft is also applied.

1.2 Overview of the methodology

- (1.2.1) The following approach is taken in order to deconstruct the problem into a manageable size for the study group:

- Analysis of a simplified point to point (P2P) model which assumes no connecting flights, in order to determine optimal bank times. Initially this is done without considering costs. Subsequently costs are included in the model.
- Calculation of viable connecting flights and their influence on demand.
- Improvement of the model to include a fixed fleet of aircraft and varying passenger demand through the day.
- Development of a timetable from the improved model.
- Considerations for including competition.

In general we assume that our timetable can be formed from scratch (i.e. there is no existing timetable from which additional constraints are present). We also assume a single aircraft type and no capacity constraints are imposed on the airport in terms of number of aircraft. The aim of posing the problem in this way as opposed to using a black box stochastic optimisation of the whole problem (which Airbus can currently do) is to allow qualitative insights into important features.

2 General approach taken

2.1 Available data

- (2.1.1) The datasets provided by Airbus includes:

- The daily passenger numbers (total demand) for each origin-destination pair in the world, which is the sum of the direct and all connecting routes actually flown in 2009. Additionally, the average fare paid for each origin-destination pair.
- The industry QSI ($\sum_{i=1}^N Q_i$) for each route and the average elapsed time for all routes involving connecting flights.

- (2.1.2) We can convert this data into a more useful format for scheduling by assuming that a flight has a fixed total turnaround time T_{OD} . In doing so, graphs such as Figure 2 can be produced showing the demanded total flight times for the P2P model.

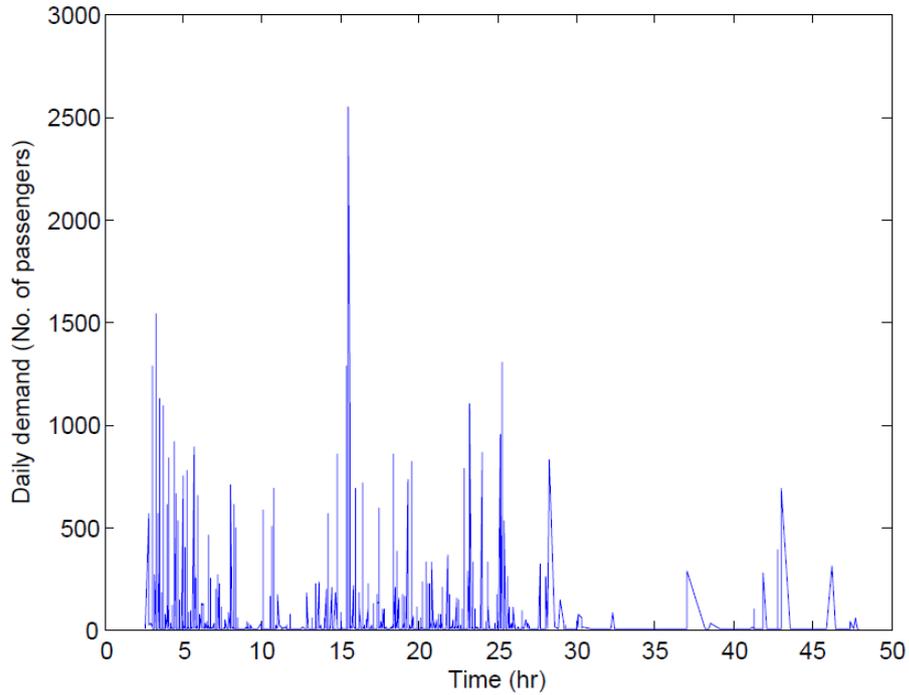


Figure 2: Plot of passenger demand against total turnaround time T_{OD} at London Heathrow (LHR). The peak just after 15hr represents flights leaving to New York from London. The medium haul category only represents flights with a total turnaround time up to 10hr.

2.2 Optimizing bank time in the P2P model

- (2.2.1) The aim is to determine the optimal *departure* bank separation time b for a given airport. In reality this can be dependent on a number of factors, however we simplify this to analysing the demand for P2P flights from a hub airport up to the medium haul bracket. Since we are not considering competitors at this stage, the demand is a measure of the revenue potential. The geography and data for Charles de Gaulle (CDG) airport is shown in Figure 3.
- (2.2.2) A penalty w_t can be applied to the time h_{OD} in hours between returning aircraft and the following departure bank (normalised by the number of flights n_{OD} for that route):

$$w_t = \sum_{OD} \frac{1}{n_{OD}} (b_f - T_{OD}), \quad b_f \geq 1. \quad (4)$$

where T_{OD} is the total turnaround time for a route and $b_f = kb$ is the next available bank time (with integer k) such that $b_f \geq T_{OD}$. The number of flights on a route is $n_{OD} = \text{floor}(D_{OD}/c)$ (if more than one flight

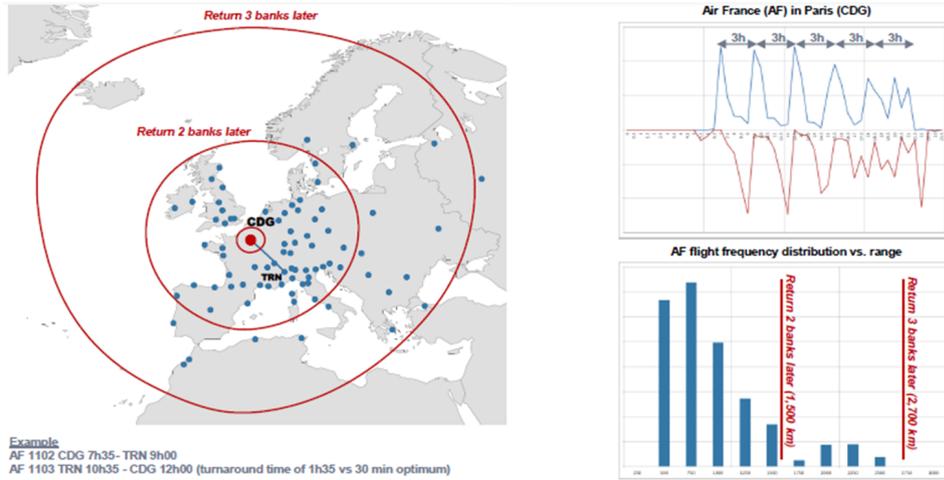


Figure 3: Air France operations from CDG.

is required to meet the demand), where c is the aircraft capacity. The weighting penalizes inefficiency since the more time aircraft spend at the hub airport between flights, the larger its value is. However, it ignores the cost associated with operating flights and is therefore the optimal bank time based solely on the geography of the hub. We would expect the optimal bank time b to occur at 1 since 1 is the minimum time between banks (in hours) allowable. Figure 4 demonstrates the penalty w_t for varying b . The large jumps in these graphs as bank time increases are due to the reduction in the total number of banks that can be operated in a single day and hence lead to a significant increase in waiting time (e.g. the opening hours for LHR are 0600 to 2200).

- (2.2.3) The solution will clearly be different if we now investigate the optimal b with respect to profitability since operating more aircraft increases costs. The profit s_{OD} for a given route is calculated as the revenue minus the cost,

$$s_{OD} = cn_{OD}f_{OD} - C_{OD}n_{OD}. \quad (5)$$

For simplicity and since the data are not available, we do not model the variation in fare price of a route based on time of departure. We also make the assumption that we have a limitless fleet of aircraft.

- (2.2.4) We can plot the profitability for each flight on a given route and then eliminate those routes which are not profitable (that are assumed not to be flown). Figure 5 shows bank time against the number of aircraft that would be required on the profitable routes to satisfy the total demand for a fixed QSI (i.e. fixed market share). The corresponding profitability plot is also shown. The optimal bank time from the model for LHR is at 4hr. In this model the number of aircraft used is equal to the number

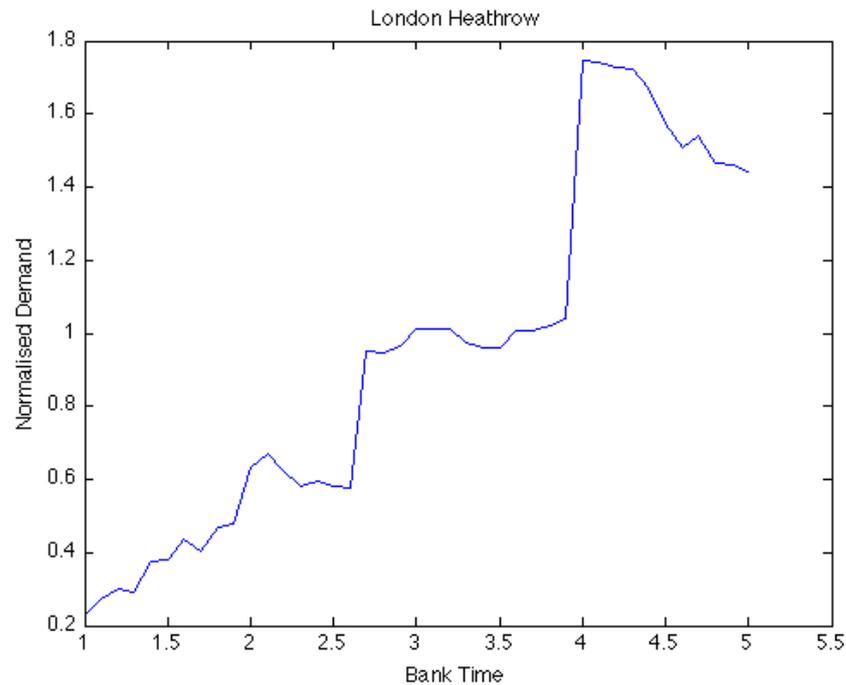


Figure 4: Bank time against normalised demand penalty w_t for LHR.

of profitable routes and therefore fixed costs are likely to be higher than in reality since aircraft can fly a route more than once. In addition, the model does not account for varying passenger demand with time of day. Therefore a better model was sought and is described later in the report.

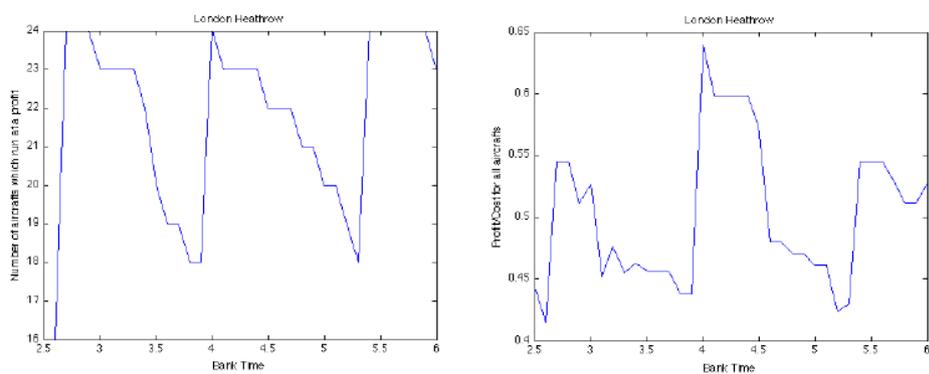


Figure 5: Left: Bank time against number of profitable aircraft flown. Right: Bank time against profitability.

2.3 Connecting flights

- (2.3.1) Of course not all routes originate from the hub therefore in the full hub and spoke model connecting flights that occur via the hub need to be represented. In order to be acceptable, a connection should not be too circuitous i.e. does not travel excessively ‘out of the way’. A circuitry ratio y can be defined as

$$y = \frac{d_{OH} + d_{HD}}{d_{OD}}, \quad (6)$$

where d_{OH} , d_{HD} and d_{OD} are distances from the origin to hub, hub to destination and origin to destination respectively. A basic model for acceptable circuitry is given by

$$y = \begin{cases} 2.7, & \text{for } x \leq 1000, \\ 2.7 - 1.3 \frac{x - 1000}{4000}, & \text{for } 1000 < x < 5000 \\ 1.4, & \text{for } x \geq 5000, \end{cases} \quad (7)$$

$$y = \begin{cases} 2.7 - 1.3 \frac{x - 1000}{4000}, & \text{for } 1000 < x < 5000 \\ 1.4, & \text{for } x \geq 5000, \end{cases} \quad (8)$$

$$y = \begin{cases} 1.4, & \text{for } x \geq 5000, \end{cases} \quad (9)$$

where x is the total distance (km) of the route. This means, for example, that a route which has a direct distance of 1000km will not be flown by a connecting route with a distance greater than 2700km. This formula is derived from empirical evidence.

- (2.3.2) Daily demand in the P2P model is increased by the presence of connecting flights. This, in turn, increases the number of profitable flights and overall profitability as we would expect. There will also be an indirect influence on the optimal bank time since large numbers of connecting passengers will affect demand for certain routes. However, examples demonstrate that this affect is minimal.

2.4 Improved model

- (2.4.1) In the previous P2P model the variation in passenger demand during the day was not considered. However, departure demand is higher at early morning and in the evenings due to business travelers. Therefore a more representative demand profile can be constructed by assuming demand is constant throughout the day ($P_{OD}/20$) with weighted pulses ($P_{OD}/10$) in the morning and evening. The time dependent ($t \in [06, 22]$) demand profile is given by the following piecewise linear function:

$$P'_{OD}(t) = \begin{cases} \frac{P_{OD}}{20}t, & \text{for } 06 \leq t \leq \tau_1, \\ \frac{P_{OD}}{20}t + \frac{P_{OD}}{10}, & \text{for } \tau_1 < t \leq \tau_2 \\ \frac{P_{OD}}{20}t + \frac{2P_{OD}}{10}, & \text{for } \tau_2 < t < 22, \end{cases} \quad (10)$$

$$P'_{OD}(t) = \begin{cases} \frac{P_{OD}}{20}t + \frac{P_{OD}}{10}, & \text{for } \tau_1 < t \leq \tau_2 \\ \frac{P_{OD}}{20}t + \frac{2P_{OD}}{10}, & \text{for } \tau_2 < t < 22, \end{cases} \quad (11)$$

$$P'_{OD}(t) = \begin{cases} \frac{P_{OD}}{20}t + \frac{2P_{OD}}{10}, & \text{for } \tau_2 < t < 22, \end{cases} \quad (12)$$

where τ_1 and τ_2 are the times at which pulses representing business passenger demand occur. The relative weighting of the pulses is an estimate in the model but can be modified to reflect reality for each individual route.

- (2.4.2) The algorithm shown in Figure 6 is implemented. A fixed number of aircraft are available from the hangar. They are sent on the most profitable routes at full capacity to meet the demand profile. Once they return after turnaround time T , they are again available to take passengers. If no profitable flight is available at a particular bank, then no aircraft is flown and we wait until the next bank. The model is run for varying bank times and aircraft numbers. Initially, a QSI based on running a single aircraft on each profitable route everyday was used. After each iteration the QSI (and hence demand) is updated to reflect the number of aircraft flown on the route. This is done until a stable QSI is reached - when there is not enough increase in demand to allow a new profitable flight.

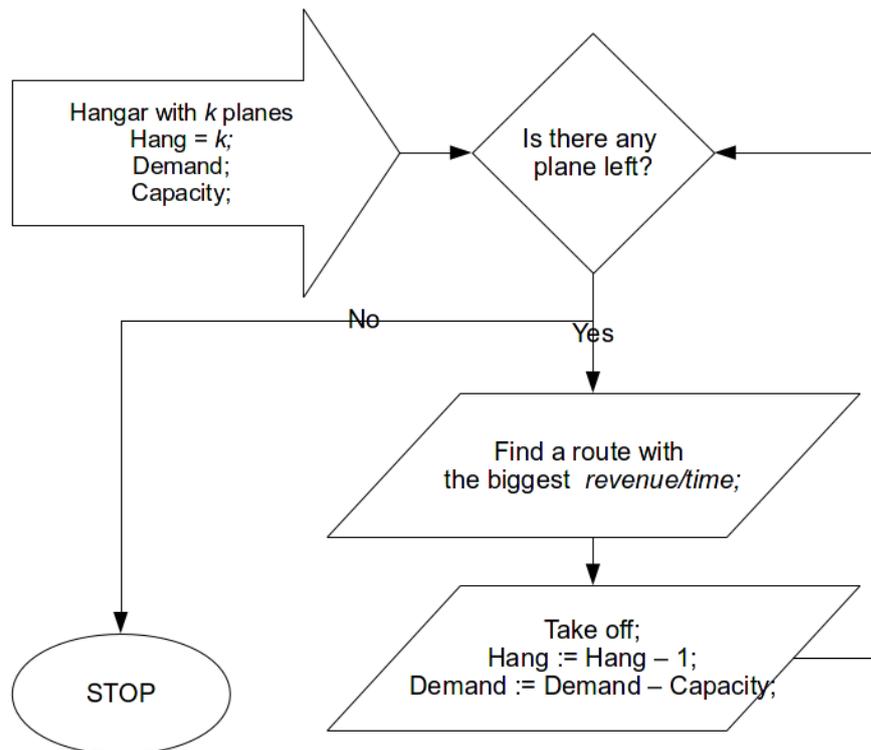


Figure 6: Algorithm for the improved model.

- (2.4.3) Using the improved model the results obtained are shown in Figure 7. The profit is maximised with six aircraft and a bank time of 5.4hr. The solution corresponds to a daily timetable given in Appendix A.2. In reality, LHR operates at near capacity and a conventional bank structure

is not observed. However, the optimal bank time given by the model is larger than expected (typically 2-4hr). We can see from the graph that profitability is extremely sensitive to bank time and therefore operating at a peak bank time would leave an airline prone to disturbances and delays.

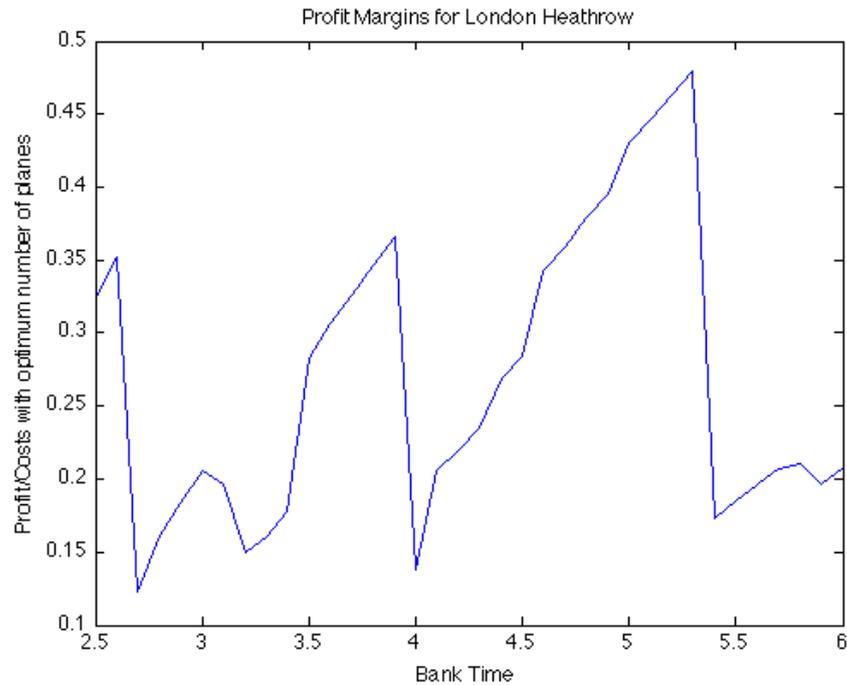


Figure 7: Bank time against profit-cost ratio for the improved model.

(2.4.4) Further improvements which can be considered but were not implemented due to time constraints are:

- In the model multiple aircraft depart at a bank which is unrealistic since aircraft cannot take-off simultaneously. A better approach would be to model the departure as a Gaussian random variable with a mean at the bank time. This framework also allows the modeling of delays. Monte Carlo simulations can then be performed to assess profitability against bank time. The impact of the variance in departure time on profitability could also be analysed and a robust timetable found.
- Other improvements can be made to the existing model including the consideration of connecting flights and long haul traffic. Long haul flights tend to arrive early in the morning or late at night coinciding with the business demand peaks whilst connecting flights would increase demand throughout the day.

2.5 Competition and game theory

- (2.5.1) Whilst there was little time in the study group to address competition, [1] is a useful starting point. In this work a passenger preference model is adopted which is a function of desired travel time, the duration and the price of the route. The penalty function w_t then becomes

$$w_t = w_1 d_{OD} + w_2 f_{OD} + w_3 |t - t'| \quad (13)$$

where d_{OD} is the distance for that route, f_{OD} is the fare, t represents the passenger's ideal departure time and t' is the actual departure time. The weights w_1 , w_2 and w_3 can be set according to market research data. Using this penalty function, the demand for route OD is shown to be a weighted logistics function (see [1] for details). Note that unlike the QSI model used previously, demand is now a function of the fare.

- (2.5.2) In [1], competition in both schedules and prices between airlines sharing a common hub is explored. This is an oversimplification since airlines operating the hub and spoke system (national carriers) have different hub airports. A brief summary of the approach is given here. The following assumptions are made:

- The set of competing airlines is known and all airlines use the same hub. Each airline can choose its flight schedule and route fare.
- Customer demand is based on (13).
- Each airline can alter its prices and schedule at any time and at no cost.

The third assumption means that each airline will choose its fares and schedules to optimize against the others' current choices, which is equivalent to a Nash equilibrium for the game where airlines choose schedules and prices simultaneously. See Appendix A.3 for a more detailed explanation of the Nash equilibrium. The third assumption, however, is in general not true since landing rights and take-off slots are not readily traded at hubs and so transactional costs are expected to be high.

- (2.5.3) It is possible to expand the game theory approach to account for additional constraints including transactional costs. Further complexity can be added by including multiple hubs. This is an approach that can be explored in more detail through further work.

3 Conclusions

3.1 Remarks

- (3.1.1) The design of aircraft timetables is a complex problem with a large number of factors to take into consideration. In the report simplified models have

been described for the hub and spoke system and some insights into the most important parameters noted. We have demonstrated how optimal bank times can be chosen by simply considering the geography of the airport, or in a more advanced model by considering the airline's market share through use of the Quality of Service Index.

- (3.1.2) The models show that profitability is highly sensitive to choice of bank time and due consideration needs to be paid to it when airlines determine schedules. The ideas presented in the study provide a starting point from which complexity can be added to the modelling.

3.2 Suggested further research

- (3.2.1) Analysis of the competition is an important factor in scheduling airline operations. The game theory approach which is discussed briefly in this report would be a good way to address this through simple hub and spoke models. However, it is important to note that computing equilibria etc. can be difficult where large numbers of competitors are present and the decision vector is of high dimensionality.
- (3.2.2) Availability and cost of take-off and landing slots at an airport are important factors in scheduling considerations and have not been considered in this study. For example, profitability might be maximized by utilizing cheaper take-off and landing slots which would lower costs, despite the lower demand for flights at these times.
- (3.2.3) Additional constraints such as airport and runway capacities play a role. In particular, they affect the assumption that departures and arrivals can occur simultaneously. It would be possible to build these into a more complex model.
- (3.2.4) To reduce complexity a fixed bank time was assumed in the study and in general this is observed at real airports. However, a varying bank structure might increase profitability and should be considered.
- (3.2.5) In a hub and spoke model arrivals from connecting flights are assumed to occur before departure banks. However, if multiple hubs are present, optimizing against another airline's schedule at a separate hub might be required and the merits of doing so should be considered.
- (3.2.6) As an aside, boarding efficiency was discussed during the study group with the general premise that boarding by blocks is less efficient than random boarding. This problem has been studied in detail and the results can be found in [3] along with simulations of the different boarding strategies.

A Appendix

A.1 Calculation of the Quality of Service Index

(A.1.1) The QSI is calculated as

$$Q = C_1 C_2 C_3 C_4, \quad (14)$$

where each component C_i is a function of a different variable. The component C_1 relates to aircraft size and for jets is given by

$$C_1 = 0.3728 + 0.00454c, \quad (15)$$

where c is the capacity of the aircraft. The component C_2 relates to whether the flight is direct or not and is given by

$$C_2 = \begin{cases} 1, & \text{for direct flights,} \\ 0.03, & \text{for connections.} \end{cases} \quad (16)$$

$$(17)$$

The component C_3 is the actual flight frequency per week i.e. if an airline flies 7 times to a destination in one week $C_3 = 7$. The final component C_4 relates to the elapsed time and is

$$C_4 = \begin{cases} 1, & \text{for direct flights,} \\ a^{-0.675}, & \text{for connections,} \end{cases} \quad (18)$$

$$(19)$$

with

$$a = \left(\frac{\text{actual elapsed time}}{\text{average elapsed time for all connections}} \right). \quad (20)$$

Recalling that the coefficients in the QSI have been calculated from large empirical data sets there are some interesting characteristics to note - connecting flights have very low demand if a direct flight is available and frequency of flights is a dominating factor.

A.2 Timetable obtained from improved model for LHR

The timetable produced by the improved model is given in Table 1. There are six operating aircraft and the bank time is 5.3hr which allows 4 banks per day.

A.3 Nash equilibrium

(A.3.1) In game theory, the Nash equilibrium is a solution concept involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep their own strategies unchanged, then the current

Destination airport code	Time of departure
BHD	0600
LIN	0600
DME	0600
NCE	0600
AMS	0600
DUB	0600
BCN	1118
BFS	1118
ORK	1118
FRA	1118
DUS	1118
GOT	1636
SVG	1636
CDG	1636
EDI	1636
KEF	1636
GLA	1636
MLA	2154
ARN	2154
MAD	2154
BHD	2154
CPH	2154

Table 1: Timetable

set of strategy choices and the corresponding payoffs constitute a Nash equilibrium. However, the Nash equilibrium does not necessarily mean the best cumulative payoff for all the players involved. In many cases all the players might improve their payoffs if they could somehow agree on strategies different from the Nash equilibrium (e.g. competing businesses forming a cartel in order to increase their profits) [2].

- (A.3.2) A formal definition of the Nash equilibrium is as follows. Let (S, f) be a game with n players, where S_i is the strategy for player i , $S = S_1 \times S_2 \dots \times S_n$ is the set of strategy profiles and $f = (f_1(x), \dots, f_n(x))$ is the payoff function. Let x_{-i} be a strategy profile for all players except for player $i \in \{1, \dots, n\}$. When each player chooses strategy x_i giving an overall strategy profile $x = (x_1, \dots, x_n)$, then player i obtains payoff $f_i(x)$. The payoff depends on the strategies of all players. A strategy profile $x^* \in S$ is a Nash equilibrium if no unilateral deviation by any single player is profitable for that player, i.e.

$$\{x^* \mid f_i(x_i^*, x_{-i}) \geq f_i(x_i^*, x_{-i}^*), x_i \in S_i, x_i \neq x_i^*, \forall i\}. \quad (21)$$

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